Meshes of Trees (MoT) and Applications in Integer Arithmetic

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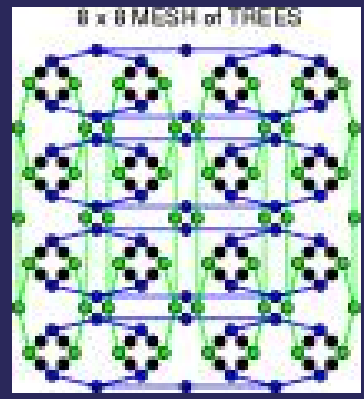


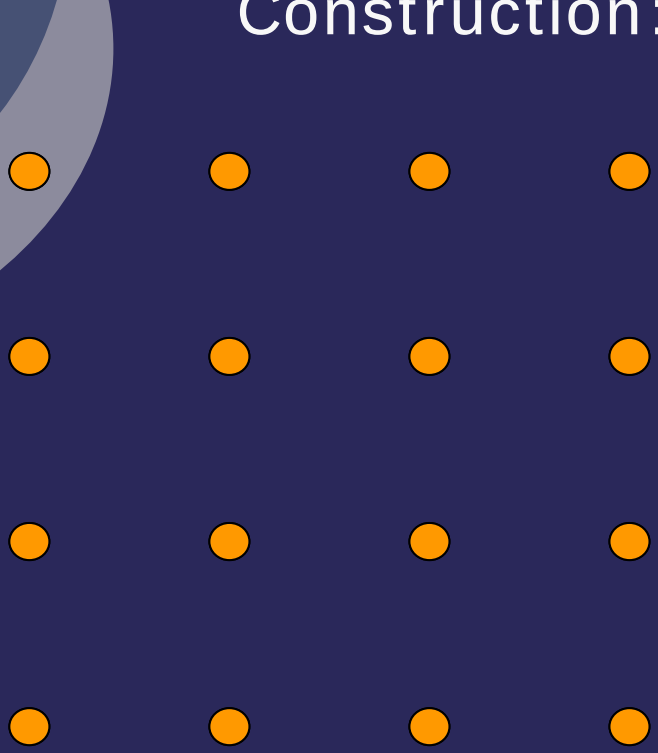
Course: Parallel Algorithms

Outline of the talk

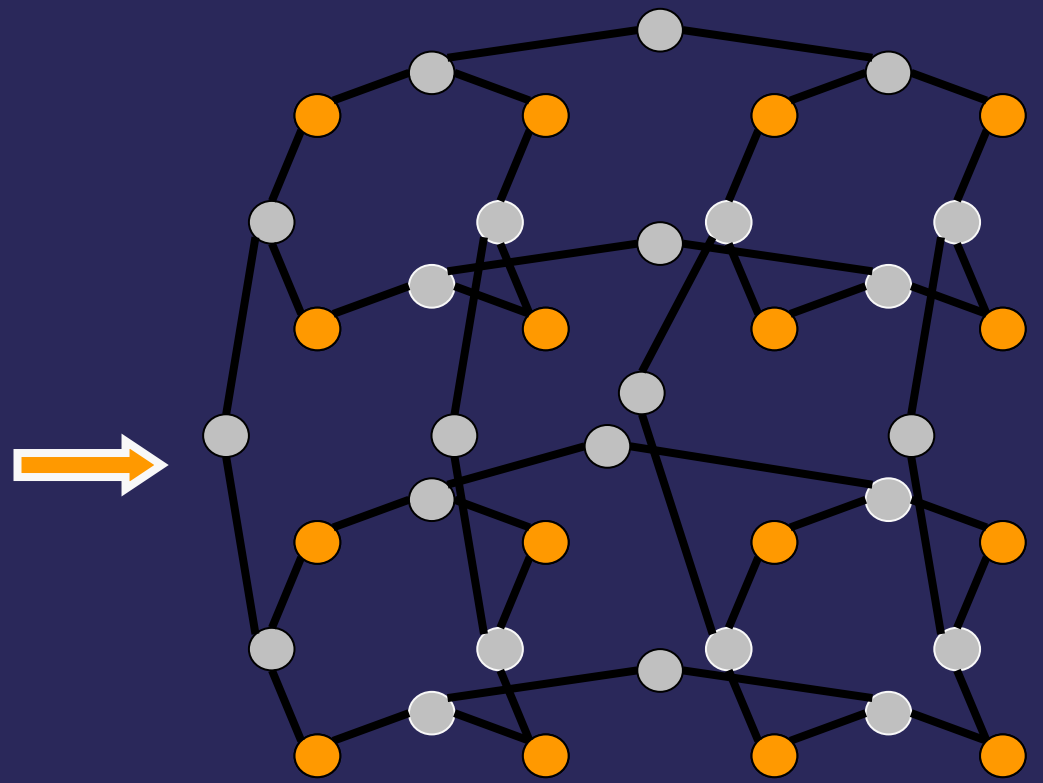
- The two-Dimensional Mesh of Trees
  - Definitions
  - Properties
  - Variations
- Integer Arithmetic Applications
  - Multiplication
  - Division
  - Powering
  - Root Finding

Definition

Construction:



$N \times N$ grid



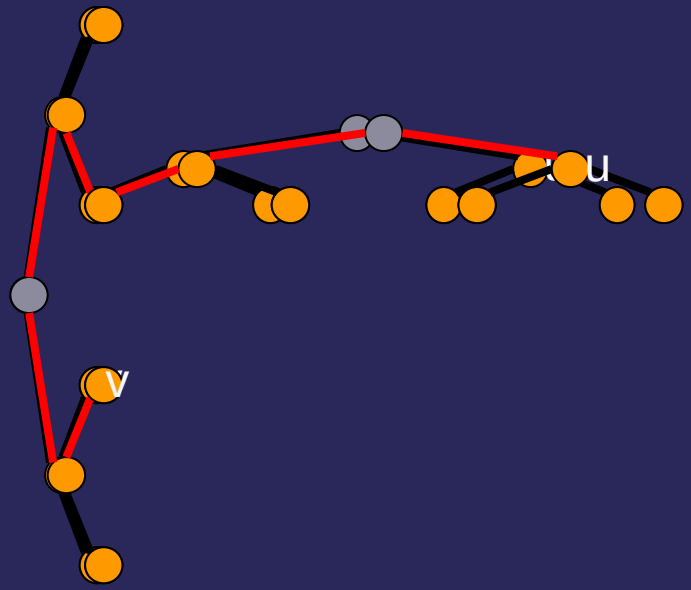
Mesh of Trees $3N^2 - 2N$ Nodes

Properties

1)Diameter (maximum distance between any pair of processors): 4logN

Proof

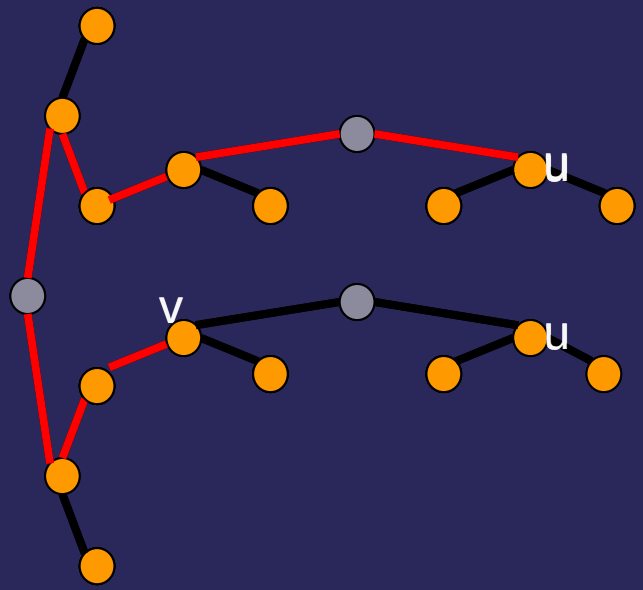
Case 1: u belongs to a row tree and v to a column tree



Dist<=2logN +2logN

Properties (cont.)

Case 2:u,v belong only to row trees (or only to column trees)



Dist=logN –r +2logN + logN + s<=4logN since r>=s

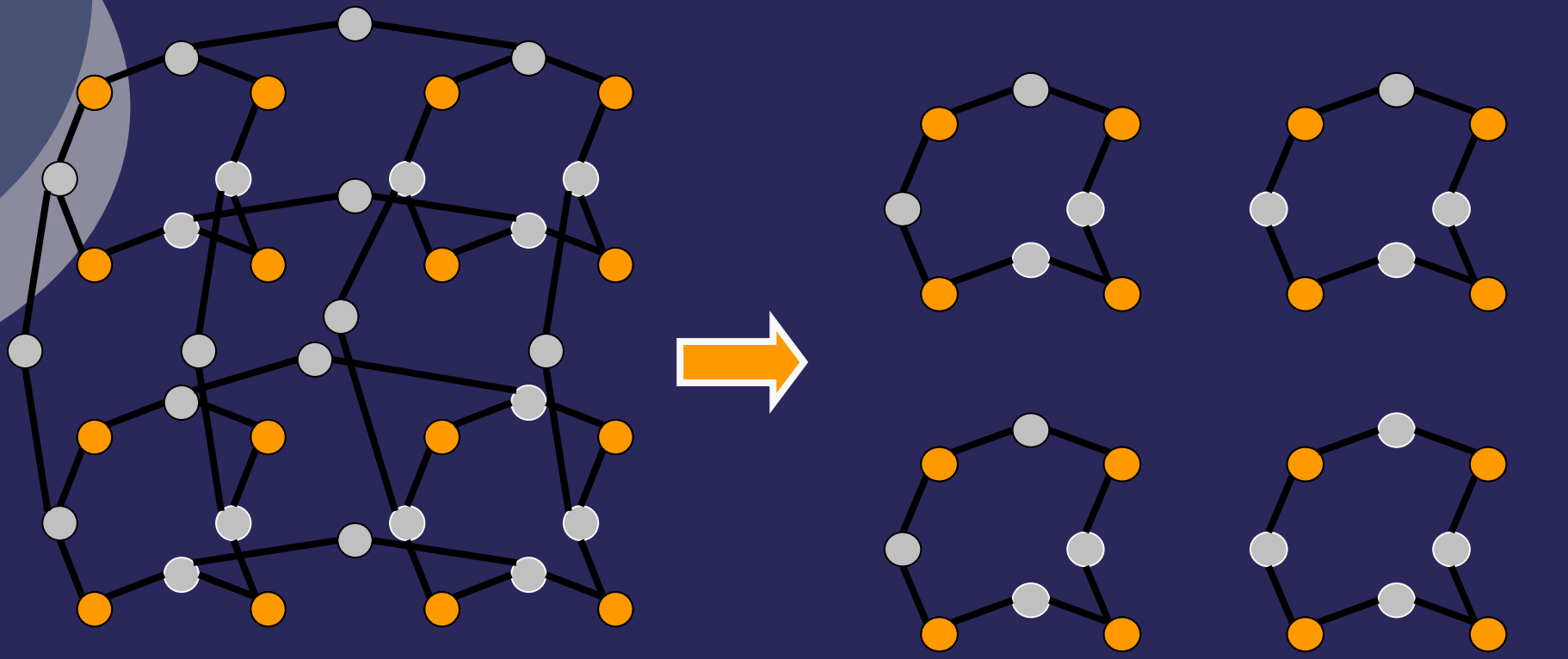
Properties (cont)

- 2)Bisection Width(the minimum number of wires that have to be removed in order to disconnect the network into two halves with “almost” identical number of processors) :N (Proof omitted)

Thus meshes of trees enjoy both **small diameter** and **large bisection width**.

This fact makes them a more efficient structure than arrays and simple trees

Recursive Decomposition

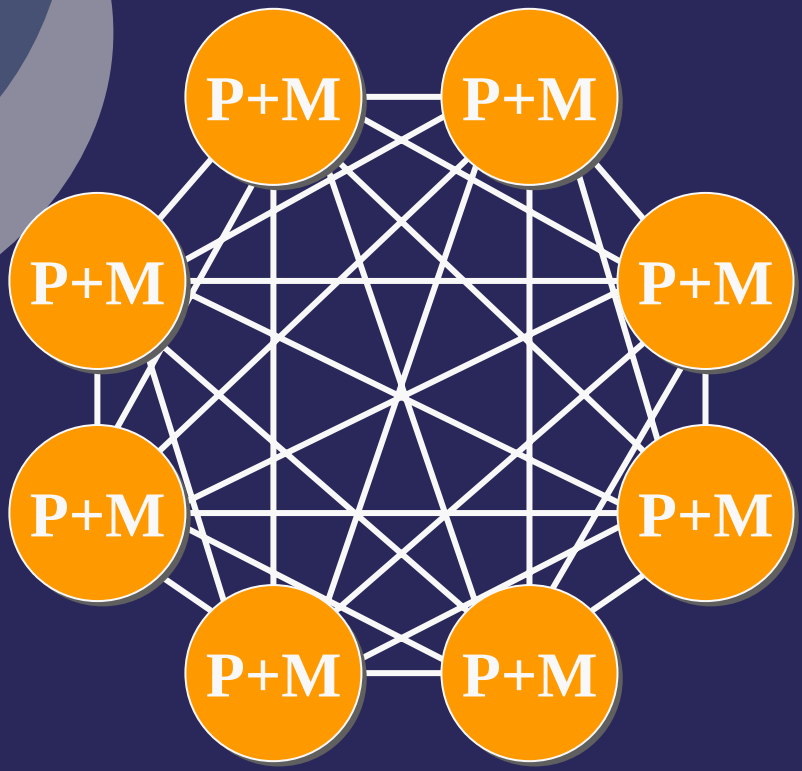


$N \times N$ Mesh of trees

Four disjoint copies of $\frac{N}{2} \times \frac{N}{2}$ Mesh of trees

Importance: This property makes mesh of trees appropriate for **recursive algorithms** for **parallel computation**

“Ideal” Parallel Computer



P: Processor

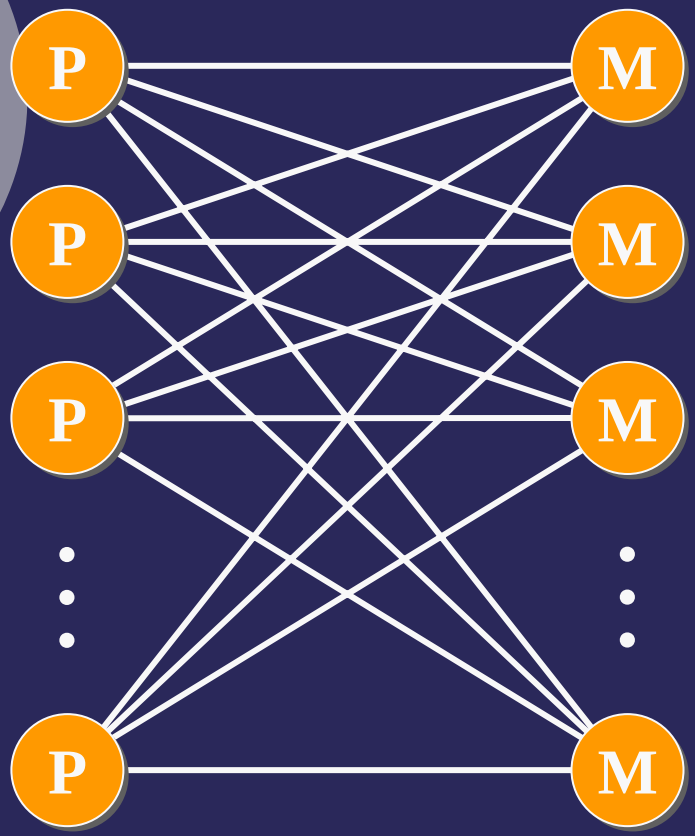
M: Memory

Every processor is linked to every other processor.

Advantage: Speed !!

Drawback: Cost

“Ideal” Parallel Computer



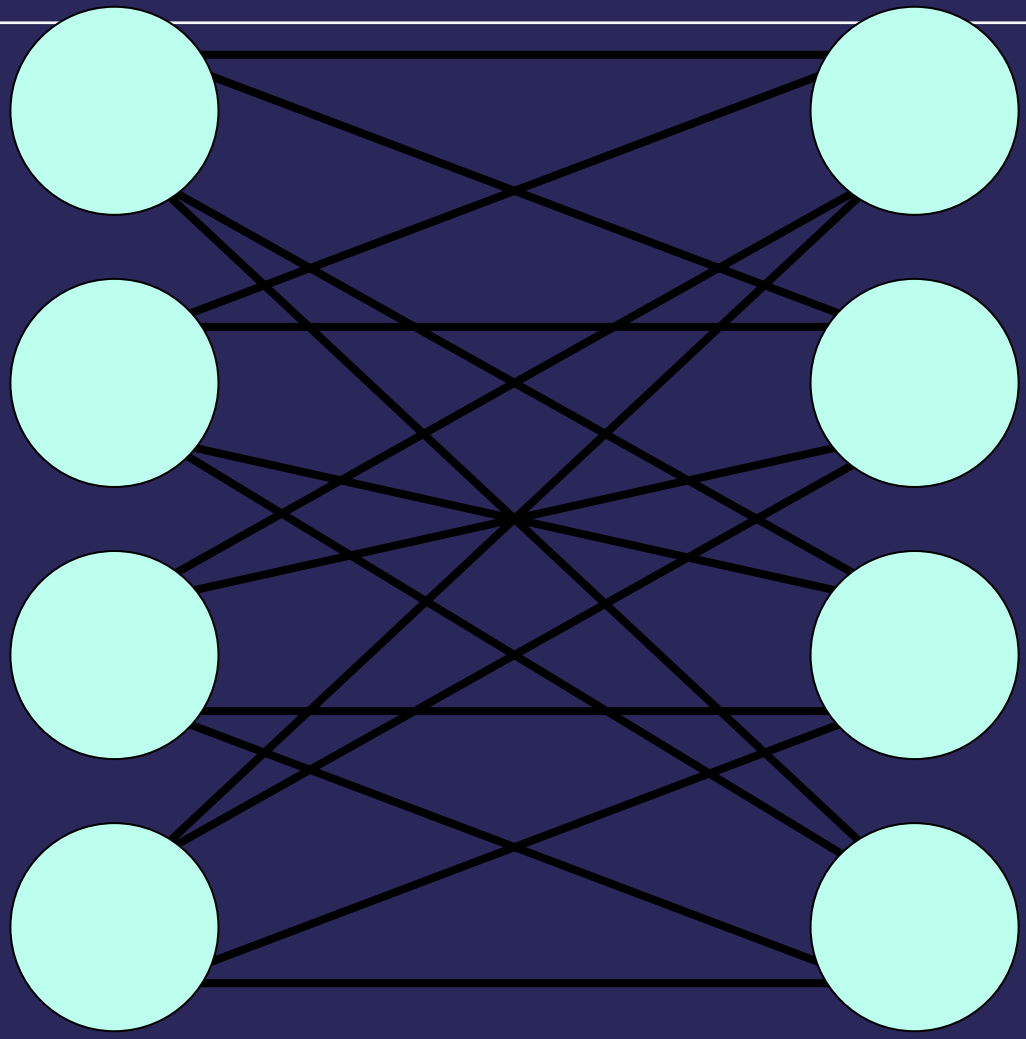
Process/Memory separation

Every Processor has direct access to a memory register

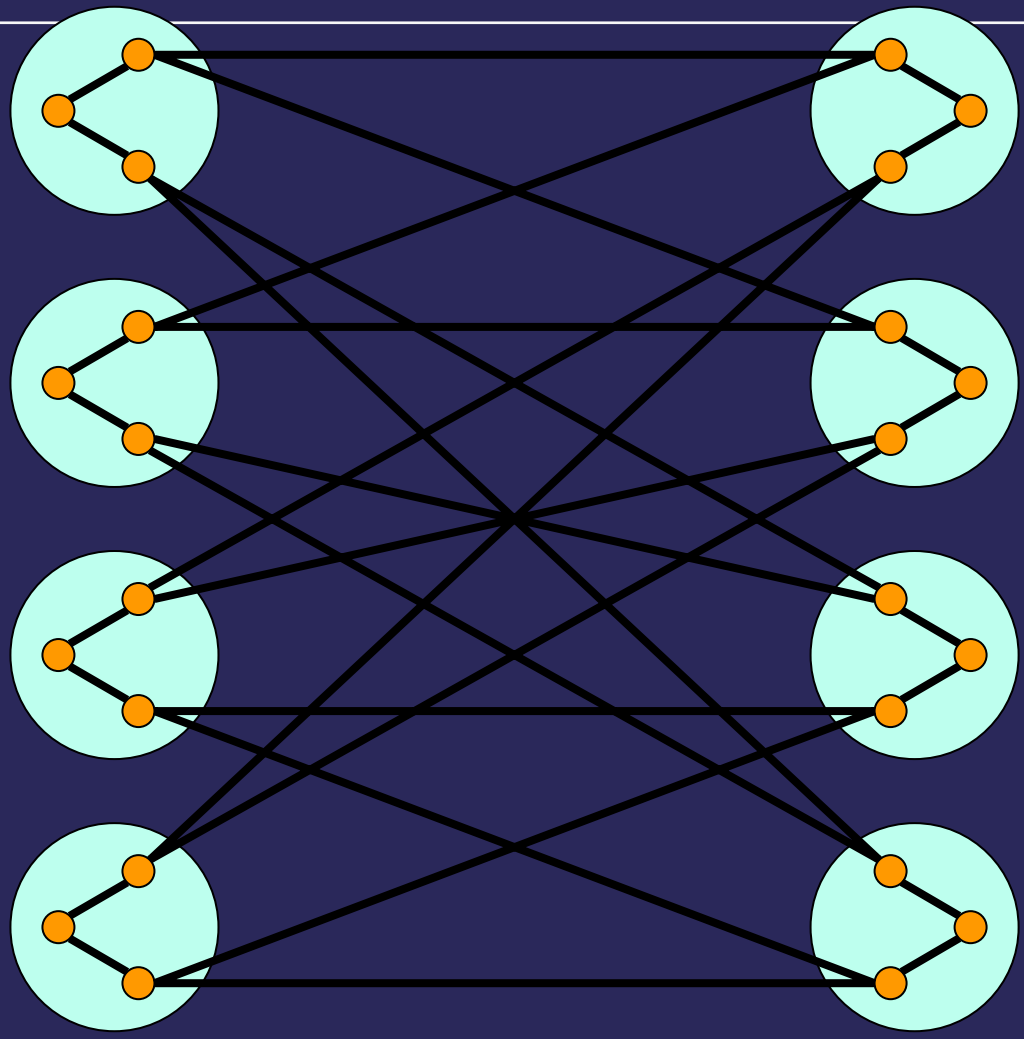
Again here the degree of each node becomes large as the number of processor increases

Idea: Why not “simulate” the complete bipartite graph?

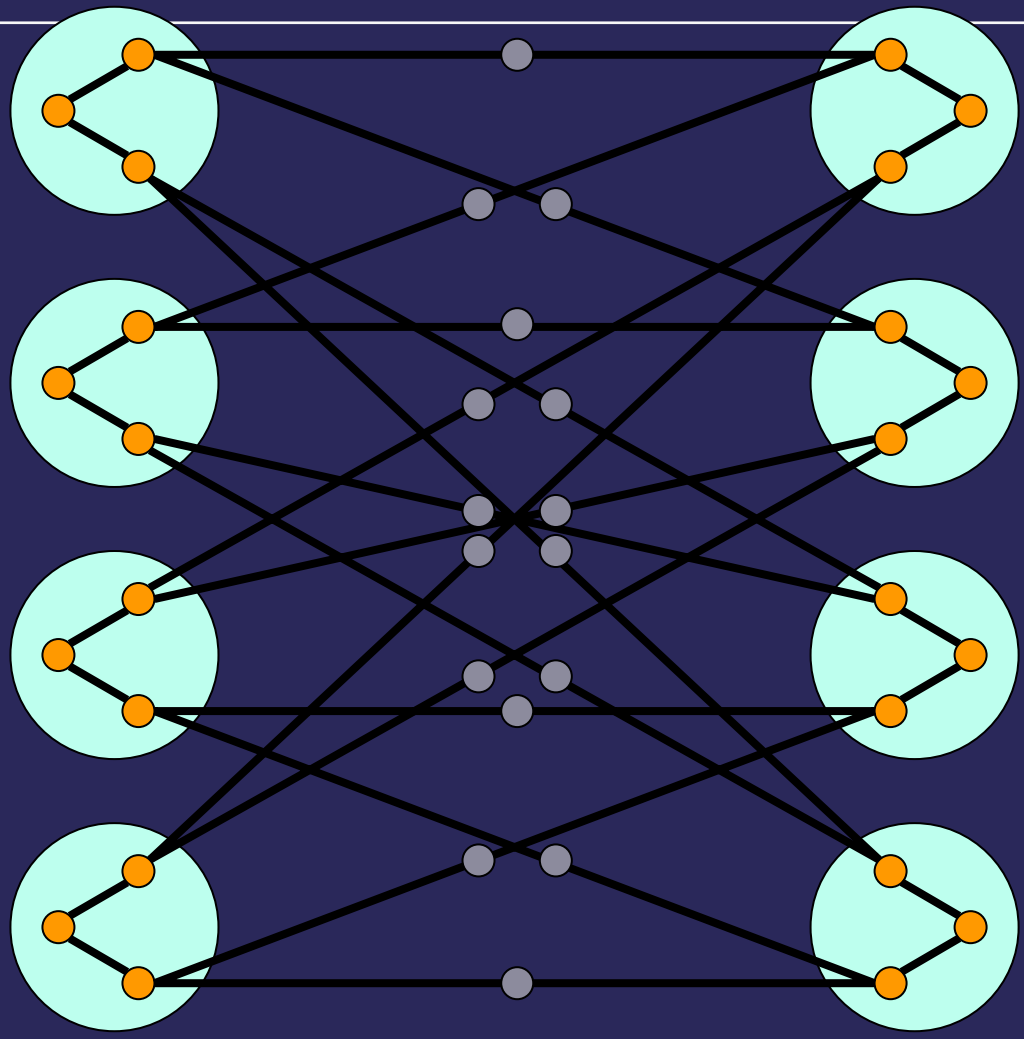
“Ideal” Parallel Computer



“Ideal” Parallel Computer



“Ideal” Parallel Computer



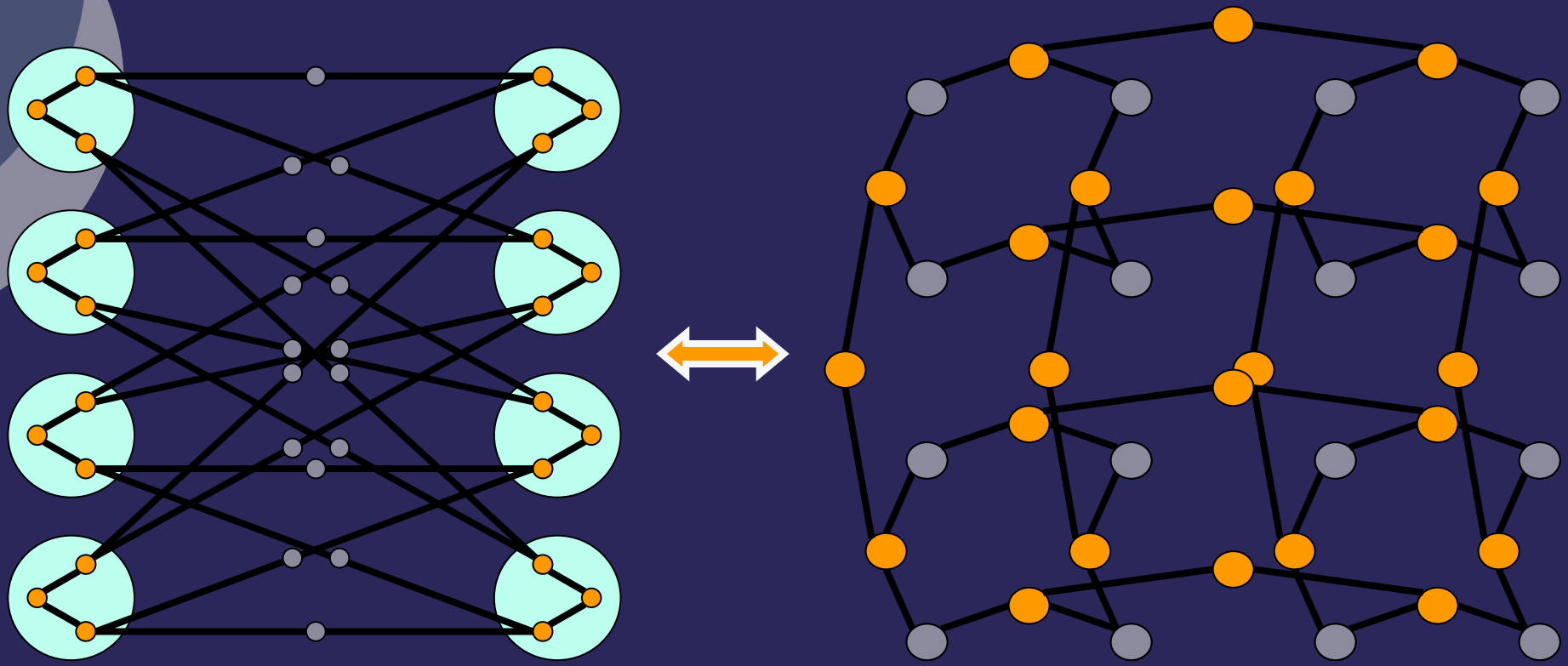
Benefits and Drawbacks

+ Simulation of any step of $K_{N,N}$ in 2logN steps
+ Bounded degree graph with essentially the computational power as $K_{N,N}$
+ We have actually constructed the NxN mesh of Trees 

- The mesh of Trees has nearly $3N^2$ nodes whereas the initial complete bipartite graph had only 2N

  Solution: **Later**

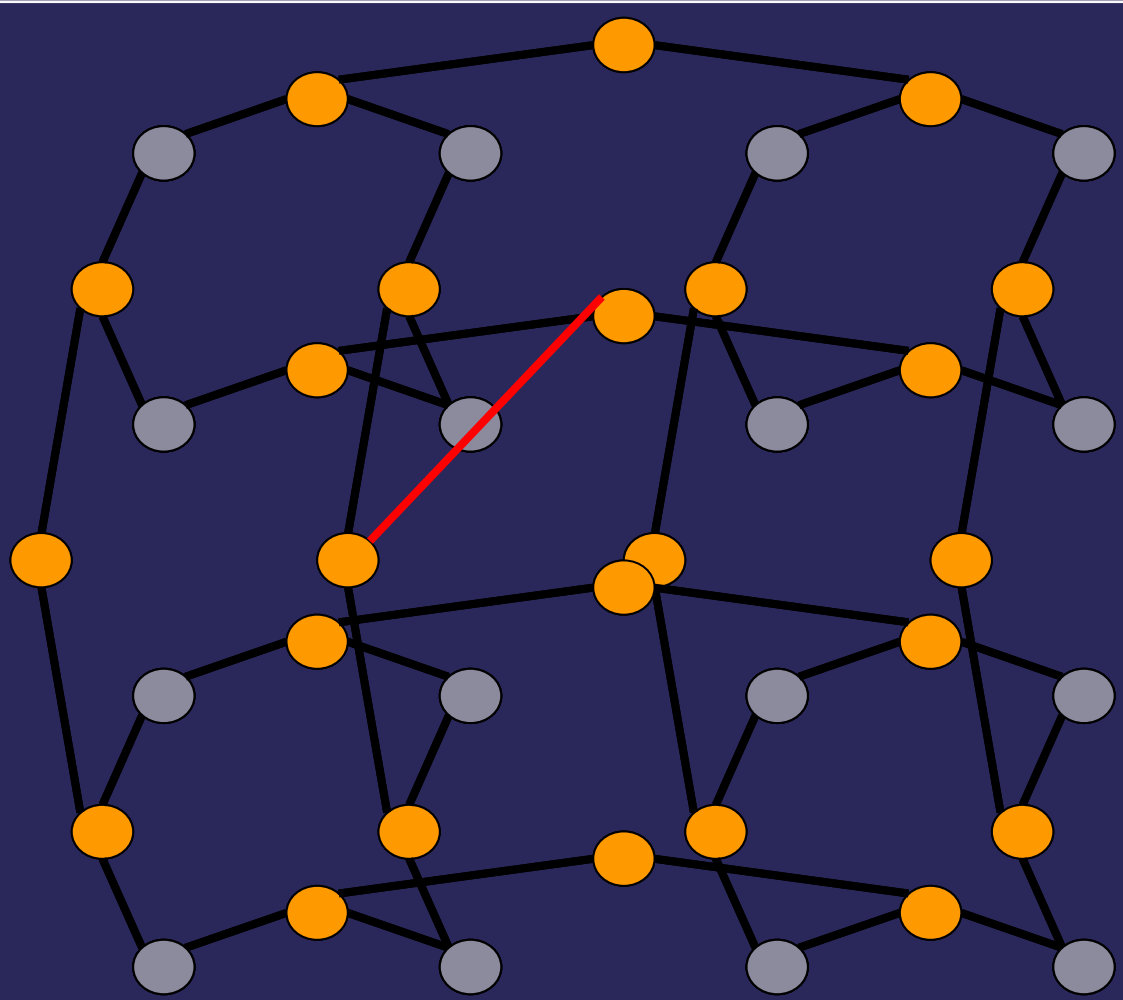
Transformation to mesh of Trees



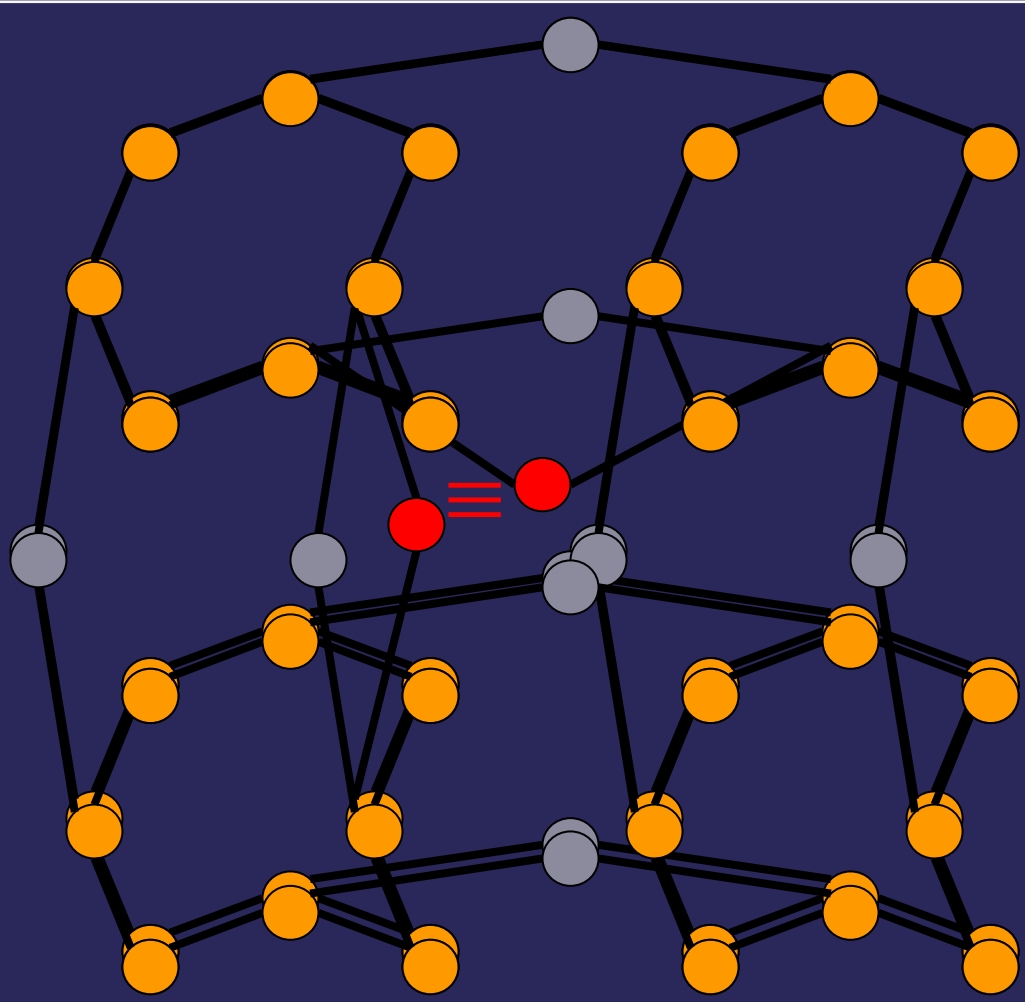
Back

Variations

1)

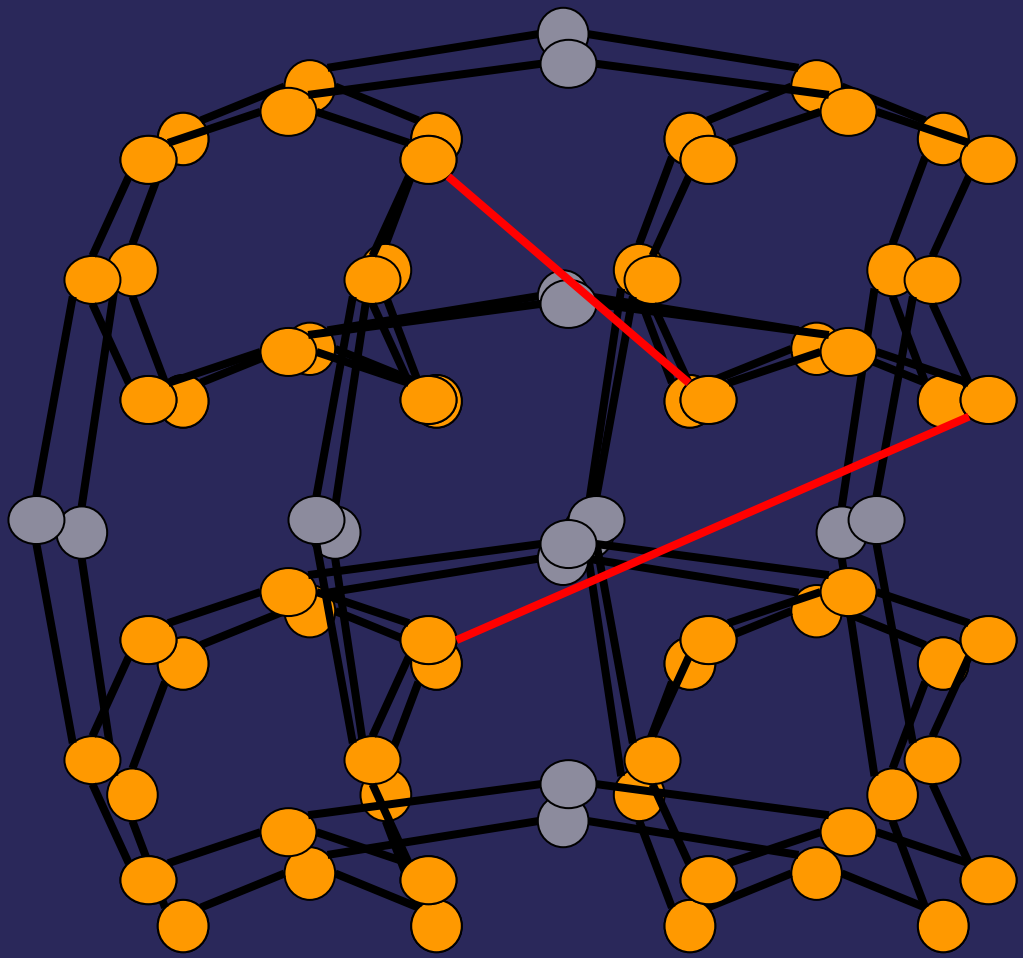


Variations (cont)



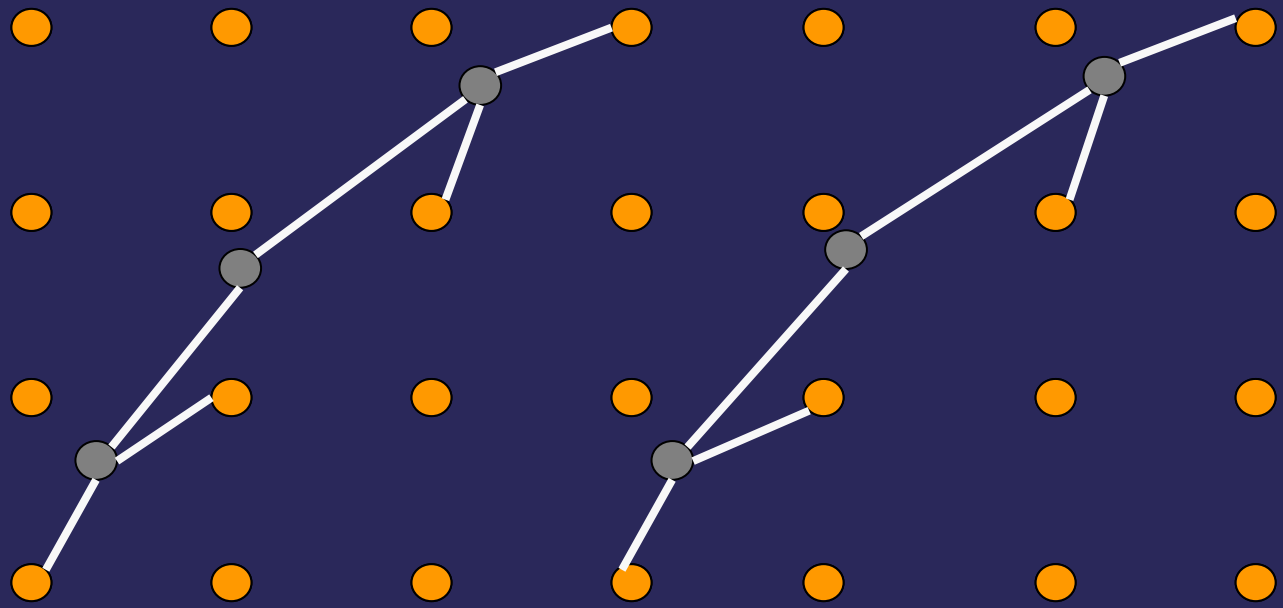
Variations (cont)

2)



Variations (cont.)

3)



Variations (cont.)

4)